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FAST TRACK COMMUNICATION

Longitudinal elliptically polarized electromagnetic waves in off-diagonal magnetoelectric split-ring composites

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Abstract

We study the propagation of plane electromagnetic waves through different systems consisting of arrays of split rings of different orientations. Many extraordinary EM phenomena were discovered in such systems, contributed by the off-diagonal magnetoelectric susceptibilities. We find a mode such that the electric field becomes elliptically polarized with a component in the *longitudinal* direction (i.e. parallel to the wavevector). Even though the group velocity $\nabla_{\mathbf{k}}\omega$ and the wavevector $\mathbf{k}$ are parallel, in the presence of damping, the Poynting vector does not just get 'broadened', but can possess a component perpendicular to the wavevector. The speed of light can be real even when the product $\epsilon\mu$ is negative. Other novel properties are explored.

(Some figures in this article are in colour only in the electronic version)

The electric field of electromagnetic waves can be polarized in different ways. For the elliptic polarization the electric fields along two orthogonal directions are 90° out of phase. In nearly all materials, such polarization is transverse in that the electric field is perpendicular to the wavevector. Very little is known if there exist materials such that the elliptic polarization is *longitudinal* in that the field rotates between a direction along the wavevector $\mathbf{k}$ and another direction perpendicular to it. In addition the consequence of this longitudinal elliptic polarization has not been explored.

Split-ring resonators, which consist of metallic rings with small cuts, have recently been of interest since they can provide negative permeability through a magnetic resonance [1]. In a composite material, when both the permittivity ϵ and the permeability μ are negative, the square of the speed of light, which is inversely proportional to $\mu\epsilon$, has a positive real part and the loss is reduced.

In this paper we study the propagation of plane electromagnetic waves through different split-ring systems consisting of arrays of rings of different orientations. We find that, for one of the modes, the square of the velocity has a positive real part even when $\mu\epsilon$ is negative; in addition, the electric field becomes longitudinally elliptically polarized.

For ordinary materials, the direction of energy flow, given by the Poynting vector, is along the direction of the wavevector. For the left-handed material, the focus is on the frequency region where the direction of the Poynting vector is opposite to the wavevector. For a longitudinal elliptically polarized wave, the Poynting vector can have a component perpendicular to the wavevector even though the group velocity is parallel to the wavevector. Other novel behaviors are also observed. We next discuss our results in detail.

The behavior of an EM wave in a material is governed by its electric and magnetic susceptibilities. In ordinary materials, the magnetization (electric polarization) is caused by an external magnetic (electric) field. However, it was recently shown [2, 3] that the split-ring system is bianisotropic and magnetoelectric—an external electric (magnetic) field can cause a magnetic (electric) polarization, that is:

$$\mathbf{M} = \hat{\alpha}_m\mathbf{B} + \hat{\alpha}\mathbf{E}, \qquad \mathbf{P} = \hat{\beta}_e\mathbf{E} + \hat{\beta}\mathbf{B}, \tag{1}$$

where $\hat{\beta} = -\hat{\alpha}^{\mathrm{T}}$ generally. Materials that are both ferromagnetic and ferroelectric at the same time exhibit this

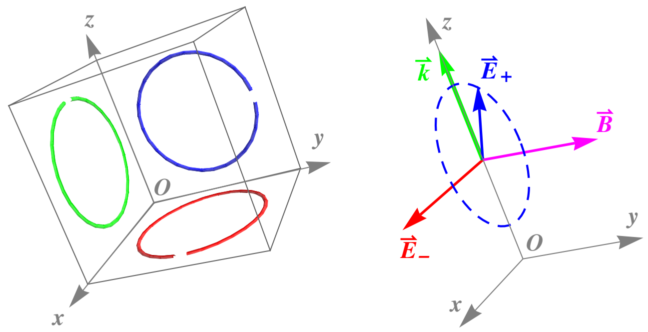

Figure 1. Illustration of the orientations of the three rings with the three cuts (left); the longitudinal elliptic polarization (E_+) and the transverse polarization (E_-) of the two normal modes found (right).

type of phenomena. Interest in this type of materials has recently been revived [4] due to improved sophistication in creating multiphase nanostructure materials. A larger magnetoelectric coefficient was observed.

Whereas the magnetoelectric effects in multiferroic materials decreases drastically above the spin wave frequency, if the rings are small enough the magnetoelectric effect in the split rings can persist up to much higher frequencies. A new aspect of the split-ring structure is that the magnetoelectric coefficients $\hat{\alpha}$, $\hat{\beta}$ are off-diagonal tensors.

While there has been much discussion of the propagation of EM waves through magnetoelectric materials where the magnetoelectric coefficients are *isotropic and diagonal* [5, 6], very little work was done to study the physics in anisotropic systems corresponding to the split-ring system. The *off-diagonal anisotropic* magnetoelectric effect of the split-ring structure is interesting in its own right.

From Maxwell's equation we get $-\nabla\times\mu^{-1}\nabla\times\mathbf{E}-4\pi\nabla\times\hat{\alpha}\partial_t\mathbf{E}/c = \partial_t^2\epsilon\mathbf{E}/c^2 - 4\pi\hat{\beta}\nabla\times\partial_t\mathbf{E}/c$, where $\hat{\mu}^{-1} = 1-4\pi\hat{\alpha}_m$, $\hat{\epsilon} = 1+4\pi\hat{\beta}_e$. We look for plane wave solutions proportional to $\exp -\mathrm{i}(\mathbf{k}\cdot\mathbf{r}-\omega t)$ and obtain

$$\mathbf{k}\times\hat{\mu}^{-1}\mathbf{k}\times\mathbf{E}-4\pi\omega\mathbf{k}\times\hat{\alpha}\mathbf{E}/c = -\omega^2\hat{\epsilon}\mathbf{E}/c^2 - 4\pi\omega\hat{\beta}\mathbf{k}\times\mathbf{E}/c. \tag{2}$$

We next discuss the solution of this equation for some examples of different split-ring systems.

(I) Three-ring medium. We first discuss the case with three types of rings with cuts along three different axes. A possible realization is illustrated in figure 1. The single split rings can also be replaced by double split rings or planar spirals, which offer us a large degree of freedom to choose the material parameters (ϵ, μ, α, etc). Each set of rings can have different 'polarities', depending on whether the cut of the ring is along the + or the − direction. We have considered systems with different polarities and found the physics to be similar. Here we consider the case when the polarities of the three kinds of rings are the same. This system is non-conventional because the magnetoelectric coefficients $\hat{\alpha}$, $\hat{\beta}$ are off-diagonal tensors. Their diagonal components and some of the off-diagonal components are zero. Systems similar to this have been studied experimentally by the Boeing group where additional arrays of wires along three orthogonal directions are also present [1]. Our calculation can be applied to such situations.

For a single split ring placed on the xy plane with a cut opened at $\phi = 0$, previous studies [2, 7, 8][3] show that an electric field E_y generates a magnetic moment m_z, while a magnetic field H_z generates an electric dipole moment p_y. For the ring structure illustrated in figure 1, it is straightforward to find the magnetoelectric tensor as

$$\hat{\alpha} = \alpha\begin{bmatrix}0 & 0 & 1\\ 1 & 0 & 0\\ 0 & 1 & 0\end{bmatrix}, \qquad \hat{\beta} = -\alpha\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ 1 & 0 & 0\end{bmatrix}.$$

For example, for a ring in the yz plane with a gap along the y direction, an electric field E_z will produce a magnetic moment m_x. This corresponds to the first row of the $\hat{\alpha}$ matrix. In the same way, the second and third rows of this matrix come from rings in the xz and xy planes with gaps along the z and x axis, respectively. After some straightforward algebra, we find that

$$\mathbf{k}\times\hat{\alpha} - \hat{\beta}\mathbf{k}\times = \alpha\begin{bmatrix}0 & k_y & -k_x\\ -k_y & 0 & k_z\\ k_x & -k_z & 0\end{bmatrix} = \alpha\mathbf{q}\times$$

where the vector $\mathbf{q} = -(k_z, k_x, k_y)$. Equation (2) thus becomes

$$\mu^{-1}[\mathbf{k}\mathbf{k}\cdot - k^2]\mathbf{E} - 4\pi\omega\alpha\mathbf{q}\times\mathbf{E}/c = -\omega^2\epsilon\mathbf{E}/c^2. \tag{3}$$

One can construct three mutually perpendicular vectors as follows. Define $\mathbf{p} = \mathbf{q}\times\mathbf{k}$ which is perpendicular to both $\mathbf{q}$ and $\mathbf{k}$. Then construct the component of $\mathbf{k}$ that is perpendicular to $\mathbf{q}$: $\mathbf{k}' = \mathbf{k} - \mathbf{q}(q\cdot k)/q^2$. In this basis $\mathbf{E} = E_k\mathbf{e}_{\mathbf{k}'} + E_q\mathbf{e}_{\mathbf{q}} + E_p\mathbf{e}_p$; equation (3) becomes $\hat{\Omega}\mathbf{E} = 0$ where

$$\hat{\Omega} = \begin{bmatrix}k'^2 + x & k_q k' & \gamma k_0 k\\ k_q k' & k_q^2 + x & 0\\ -k_0 k\gamma & 0 & +x\end{bmatrix}, \tag{4}$$

$x = k_0^2 - k^2$, $k_q = \mathbf{e_q}\cdot\mathbf{k}$, $k_0 = (\mu\epsilon)^{1/2}\omega/c$, $\gamma = 4\pi\alpha\sqrt{\mu/\epsilon}$. After some algebra, we find that

$$\det(\Omega) = k_0^2[x^2 + (\gamma k)^2(k_q^2 + x)].$$

The condition $\det(\Omega) = 0$ leads to a quadratic equation in x and we obtain the dispersion

$$k_0^2 = k^2 - 0.5((\gamma k)^2 \pm [(\gamma k)^4 - 4(\gamma k k_q)^2]^{1/2}). \tag{5}$$

We next explore the implication of this result.

The physics is particularly simple for $k_q = 0$, or close to a resonance when γ becomes large and the term proportional to k_q can be neglected. In that case from equation (4) the electric field along $\mathbf{q}$ is not coupled to the other two components. For example, the condition $k_q = 0$ is obtained if $\mathbf{k}$ is along one of the axis, e.g. $k_x = k_y = 0$, the directions $\mathbf{k}'$, $\mathbf{q}$, $\mathbf{p}$ then correspond to the directions $\mathbf{z}$, $-\mathbf{x}$, $\mathbf{y}$, respectively. In this limit, the dispersion for the two normal modes is given by $\omega = vk$ with

$$v^2 = c^2/(\mu\epsilon), \qquad v^2 = c^2[1/(\mu\epsilon) - (4\pi\alpha)^2/\epsilon^2]. \tag{6}$$

The first mode corresponds to a − sign in equation (5) and is one that we normally expect. The electric field is

[3] This paper calculates a series of constants A_m. In the cgs units used in this paper, $L_m = 0.25(A_{m+1} + A_{m-1})/c^2$.

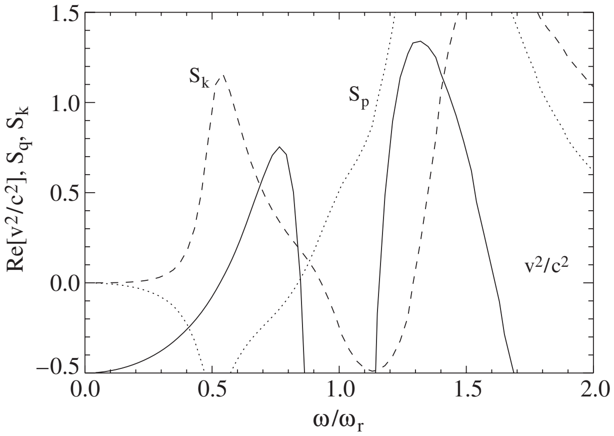

Figure 2. The transverse and longitudinal Poynting vectors S_p (dotted line) and S_k (dashed line) normalized by $c|E_p|^2/(4\pi)$ and the square of the velocity (solid line) as a function of the frequency normalized by the resonance frequency. $R^3/V = 0.03$, $\epsilon = -2$, the conductor resistance $2\pi R r_c = 0.1 Z_0$, where $Z_0 = (\mu_0/\epsilon_0)^{1/2} = 377\ \Omega$ is the impedance of the vacuum.

transverse and polarized along q. This is illustrated by the field E_- in figure 1. The second mode in equation (6) behaves quite anomalously and is the key discovery of the present work. Because α possesses a significant imaginary part [9][4], the factor $-(4\pi\alpha)^2/\epsilon^2$ provides for an increase in the group velocity if ϵ is real. In most experiments there is a background of metallic wires. We assume that this is the dominant contribution to the dielectric constant ϵ, which can then become negative throughout the frequency regime that we consider. Even when $\mu\epsilon$ is negative, the second factor can render the real part of ω^2 positive close to resonance. This can be seen in figure 2 where we have evaluated $(v/c)^2$ using the value of α we recently calculated [8, 7, 9] (see footnotes 3 and 4). On both sides of the resonance where μ changes sign, the real part of v^2 remain positive. Very close to the resonance, the real part of α becomes large. Even though the product $\mu\epsilon$ is positive, Re[v^2] can be less than zero, as is shown in figure 2.

For the anomalous mode (second mode in equation (6)), we get from equation (4) that $\mathbf{E} = E_k\mathbf{e}_k + E_p\mathbf{e}_p$; $E_k/E_p = -\gamma k/k_0$. Because γ possesses a significant imaginary part the two components are 90° out of phase; the electric field is *longitudinally* elliptically polarized in the plane formed by **p** and **k**! This is illustrated by E_+ in figure 1. The magnetic field $\mathbf{B} = \mathbf{k}\times\mathbf{E}c/\omega = ckE_p\mathbf{e}_q/\omega$ is transverse.

In general, for a non-zero k_q, from equation (4), we get $E_q = -E_{k'}k_q k'/(k_q^2+k_0^2-k^2)$ and $E_p = -E_{k'}k_0 k\gamma/(k_0^2-k^2)$. The electric field has a component in the k–q plane and another component that is 90° out of phase in a direction perpendicular to the k–q plane. Thus it is also longitudinally elliptically polarized.

In previous studies a key question is the direction of energy flow, which is given by the Poynting vector $\mathbf{S} = \mathrm{Re}[c\mathbf{E}\times\mathbf{H}^*/(4\pi)]$. Kamenetskii [10] has shown that, for bianisotropic media, Poynting's theorem has the continuity equation form and energy transport is possible if the envelope function of the wavepackets satisfy certain conditions. Because of the longitudinal elliptic polarization, both **E** and **H** now have components $E_\parallel$, $H_\parallel$; $E_\perp$, $H_\perp$ along and perpendicular to **k**, the cross-product of **E** and **H** can have contributions Re[$E_\parallel H_\perp^*$], Re[$E_\perp H_\parallel^*$] perpendicular to **k**. When damping is included, $E_\parallel$ ($E_\perp$) and $H_\perp$ ($H_\parallel$) contain contributions that are out of phase with each other and thus give rise to a non-zero component S_p of the Poynting vector that is perpendicular to the wavevector. This becomes significant close to the resonance when the real part of α becomes large, as is illustrated in figure 2 where we show the transverse and the longitudinal components S_p and S_k.

Photonic crystals and anisotropic materials exhibit negative refraction because the group velocity $\mathbf{v_g} = \nabla_\mathbf{k}\omega$ is not parallel to the wavevector. The perpendicular direction of energy flow discussed here does not come from the group velocity, which is along the wavevector for the example discussed here. Differentiating equation (5) we get $2k_0\mathbf{v_g}/c = 2k\mathbf{e}_k - \gamma^2 k\mathbf{e}_k \pm [\gamma^4 k^3\mathbf{e}_k - 2\gamma^2(kk_q^2\mathbf{e}_k + k^2k_q\mathbf{e}')]/[(\gamma k)^4 - 4(\gamma k k_q)^2]^{1/2}$, where $\mathbf{e}' = -(k_z+k_y, k_x+k_z, k_x+k_y)/k$. For $k_q = 0$, the coefficient of the $\mathbf{e}'$ term is zero and the group velocity is along the wavevector.

Our effect is present only in the presence of damping, when there is no exact theorem that requires the Poynting vector to be parallel to the group velocity. Usually we expect a smearing in the direction of energy flow in the presence of damping. Here we find the presence of additional terms in the perpendicular direction.

(II) Two-ring medium. We next consider the case with arrays of two types of rings, one with rings in the xz plane and the other one with rings in the xy plane; the cuts of *both* rings are along the x axis.

For the present case $\mu_{zz} = \mu_{yy} = \mu$, $\mu_{xx} = 1$, $\alpha_{yz} = -\alpha_{zy} = \alpha_0$ and $\beta_{zy} = -\beta_{yz} = -\alpha_0$. Experimental systems often include an array of wires along the x direction [1]. The orientation of the rings is as illustrated in figure 1 where the rings in the yz plane are now absent. We have studied two cases: (1) the dielectric constants are diagonal, so that along the x axis they are different, $\epsilon_{xx} = \epsilon$, $\epsilon_{yy} = \epsilon_{zz} = 1$ and (2) the dielectric constants are diagonal and of the same value ϵ. We find that in the expression for the dispersion α occurs in combination with the *first* power of ϵ for case (1) but with the *second* power of ϵ for case (2). To illustrate the essential physics, we focus on case (1) in this paper, where the anisotropic dielectric constant can come from additional contributions from metallic wires that are present in many experimental systems.

The solution for this case is mathematically similar to case (I) with three types of rings. The details of this will thus be left

[4] The susceptibilities are given by $\alpha_m = -f\chi_0\pi/(2c^2)$, $\beta = ic\alpha_m/(\omega R)$, $\beta_e = -\chi_0\pi f/(R\omega)^2$ and $\alpha = -\beta$, where $f = R^3/V$ is the filling factor, $\chi_0^{-1} = (L_0+L_1/2)[1-(\omega_r/\omega)^2]-1.5ir_c/\omega$, L_m, L'_m, C_m and C'_m are the mth Fourier component of the self-and mutual inductances and capacitances given in [6]. We have used a more sophisticated 'Clausius–Mosotti' approximation of $\mu = (1+8\pi\alpha_m/3)/(1-4\pi\alpha_m/3)$ for the composite. $\omega_r = [(1/C_1 + ir_c)/(2L_0+L_1)]^{0.5}$, V is the volume per ring and r_c is the resistance of the ring. As can be seen β_{yz} is mostly imaginary. For the dielectric constant, we have assumed that there is a background produced by metallic wires of volume fraction f' with dielectric constant ϵ_m so that the total dielectric constant is of the order of $\epsilon = f'\epsilon_m + 4\pi f\beta_e$ with the dominant contribution from ϵ_m.

to a longer publication. We obtain[5]

$$k_0^2 = k_z^2 + k_y^2 + k_x^2\mu^{-1}, \qquad k_0^2 = v^2(k_z^2 + k_y^2) + k_x^2/\mu \quad (7)$$

where $v^2 = v_0^2 - \alpha'^2/\epsilon$. $k_0 = \omega/c$, $\alpha' = 4\pi\alpha$ and $v_0^2 = 1/(\mu\epsilon)$ is the group velocity when the magnetoelectric effect is absent. We have verified this result by a direct brute force computation of the eigenvalue equation in the original basis using a symbolic manipulation program to handle the algebra.

For the second mode, the velocity v now contains a term $-\alpha'^2/\epsilon$. Since α possesses a significant imaginary part, if ϵ is positive, this term can increase the speed of light and make the speed real even if μ is negative. If ϵ is negative, this term will now lower the speed of light, the opposite to case (I)!

For the first mode, the electric field is along $\mathbf{e_q}$ which is perpendicular to both $\mathbf{k}$ and $\mathbf{e}_x$. For the second mode, the electric field is perpendicular to $\mathbf{e}_q \propto (0, -k_z, k_y)$: $\mathbf{E} = E_k\mathbf{e}_k + E_p\mathbf{e}_p$, where $\mathbf{e_p} = \mathbf{e_k} \times \mathbf{e_q}$:

$$E_p/E_k = [k_x^2(\epsilon - 1)/k^2 + 1]k_0/[q\alpha' - k_0(\epsilon - 1)p_xk_x/(kp)].$$

The electric field now has a *longitudinal* component along the direction of the wavevector. Because α is mostly imaginary the longitudinal and the transverse components of the electric field are now out of phase: the electric field is elliptically polarized.

The new kind of longitudinal elliptic polarization has other unexpected consequences. For example, we find by full-wave numerical computations that when a 'TE'-polarized two-dimensional (2D) Gaussian beam[6] with the $\mathbf{E}$ field parallel to the interface is incident from air on the two-ring medium, the refracted wave inside the medium splits into two beams[7], as depicted in figure 3(a). One of the beams corresponds to the normal mode with the $\mathbf{E}$ field perpendicular to the $\mathbf{k}$ vector, as schematically illustrated in figure 3(c). The other beam corresponds to the anomalous mode with $\mathbf{E}$ field rotating on the $\vec{x}$–$\vec{k}$ plane, as schematically shown in figure 3(b).

The anomalous refraction can be understood from the boundary conditions that the tangential components of $\mathbf{E}$ and $\mathbf{H}$ are continuous. Because the anomalous refracted beam is longitudinal elliptically polarized with a $\mathbf{E}$ component lying on the $\mathbf{x}$–$\mathbf{k}$ plane, a TE-polarized incident wave with the $\mathbf{E}$ field along $\mathbf{x}$ will generate a longitudinal elliptically polarized wave with a y component; the continuity of the total tangential $\mathbf{E}$ can only be satisfied if an additional wave is present also with its $\mathbf{E}$ field in the yz plane.

In conclusion, we find that, for a collection of split rings, because of its anisotropic off-diagonal magnetoelectric tensor, many interesting new phenomena remain to be discovered, with or without additional arrays of wires. Examples of these are (a) onset of transmission even for a negative μ, or no transmission even when both ϵ and μ are negative, (2) perpendicular transport of light and (3) anomalous behaviors in refraction and reflection. This paper provides for directions of the conditions under which such phenomena can be observed. Similar results can be obtained for the case with only one type of ring and will be discussed in a fuller publication.

[5] The magnetic susceptibility is now anisotropic. By direct computation, $\mathbf{k} \times \hat{\mu}^{-1}\mathbf{k} \times \mathbf{q} = -(k_z^2 + k_y^2 + k_x^2/\mu)\mathbf{q}$, $\mathbf{k} \times \mu^{-1}\mathbf{k} \times \mathbf{k} = 0$, $\mathbf{k} \times \mu^{-1}\mathbf{k} \times \mathbf{p} = -(k^2/\mu)\mathbf{p}$.

[6] For the definition of a two-dimensional Gaussian beam, see, for example [11].

[7] In our calculations, we first expand the 2D Gaussian beam into plane waves, then solve the Maxwell equations for each plane wave component, and finally sum up the results for all components.

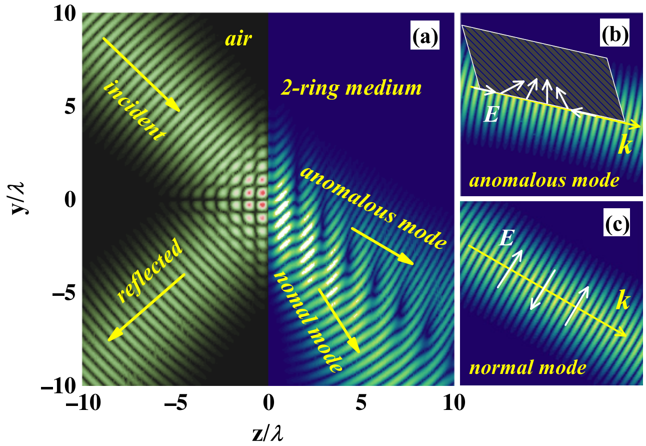

Figure 3. (a) Wave reflection/refraction as a 2D 2λ-wide Gaussian beam with a TE polarization ($\vec{E}||\vec{x}$) on a two-ring medium with $\epsilon_{xx} = 10$, $\epsilon_{yy} = \epsilon_{zz} = 2$, $\mu_{xx} = 1$, $\mu_{yy} = \mu_{zz} = 1.1$, $\alpha_0 = 5i/(4\pi Z_0)$. Expanded views of (b) the anomalous beam with $\mathbf{E}$ vector rotating on the $\vec{x}$–$\vec{k}$ plane and (c) the normal beam with $\mathbf{E}$ perpendicular to $\vec{k}$.

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References

[1] Pendry J B 2000 *Phys. Rev Lett.* **85** 3966
Shelby R A, Smith D R and Schultz S 2001 *Science* **292** 79
Ramakrishna S A 2005 *Rep. Prog. Phys.* **68** 449–521
O'Brien S, McPeake D, Ramakrishna SA and Pendry J B 2004 *Phys. Rev.* B **69** 241101
Movchan A B and Guenneau S 2004 *Phys. Rev.* B **70** 125116
Guenneau S and Zolla F 2007 *Physica* B **394** 145–7
Milton G and Willis J R 2007 *Proc. R. Soc.* A **463** 855
[2] Marques R, Medina F and Raffi-El-Idrissi R 2002 *Phys. Rev.* B **65** 144440
[3] Katsarakis N, Koschny T, Kafesaki M, Economou E N and Soukoulis C M 2004 *Appl. Phys. Lett.* **84** 2943
[4] For some recent references, see, for example Wan J G, Liu J M, Wang G H and Nan C W 2006 *Appl. Phys. Lett.* **88** 182502
Jiang *et al* 2004 *Science* **302** 661
[5] Pendry J 2004 *Science* **306** 1353
[6] Lindell I V, Sihvola A H, Tretyakov S A and Vitanan A J 1994 *Electromagnetic Waves in Chiral and Bi-Isotropic Media* (Boston, MA: Artech House)
[7] Zhou L and Chui S T 2007 *Appl. Phys. Lett.* **94** 041903
[8] Zhou L and Chui S T 2006 *Phys. Rev.* B **74** 035419
[9] Chui S T, Zhang Y and Zhou L 2008 *J. Appl. Phys.* **104** 034305
[10] Kamenetskii E O 1996 *Phys. Rev.* E **54** 4359
[11] Kriezis Em E, Pandelakis P K and Papagiannakis A G 1994 *J. Opt. Soc. Am.* A **11** 630